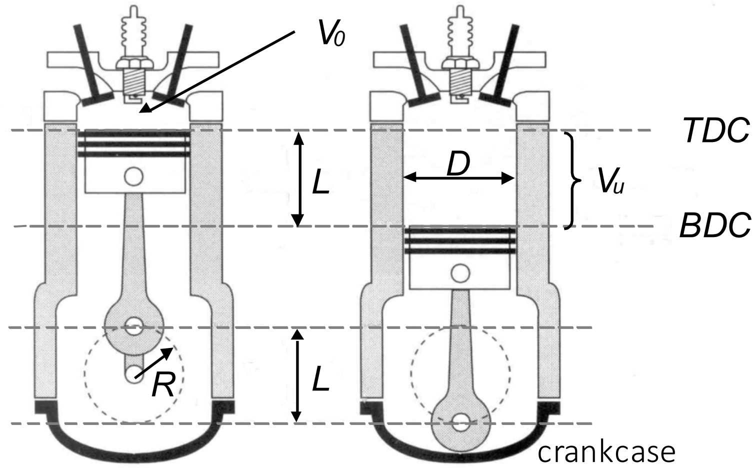
Formula memento
Engines

1. Operating principle



D : Bore (***D***iameter)

L : Stroke (***L***ength of stroke)

R : crank radius ⇒ $R = \frac{L}{2}$

TDC : Top Dead Center

BDC : Bottom Dead Center

V_0 : clearance volume = 'dead' volume ⇒ $V_0 = V_{TDC}$

V_U : displaced volume = swept volume by 1 piston ⇒ $V_u = L \cdot \frac{\pi \cdot D^2}{4}$

V_C : maximum cylinder volume (=displaced + dead volume) ⇒ $V_C = V_{BDC} = V_0 + V_u$

n : number of cylinders

V_{cyl} : engine displacement (=volume swept by all cylinders) ⇒ $V_{cyl} = n \cdot V_u$

ε : geometrical compression ratio ⇒ $\varepsilon = \frac{V_C}{V_0} = \frac{V_u + V_0}{V_0}$

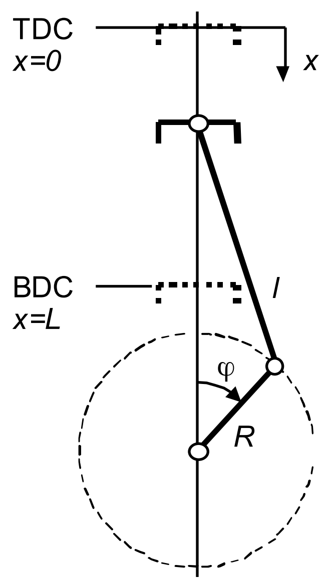
φ : crank angle ($\varphi_{c.a}$) = time (motion)

l : connecting rod length

λ : ratio of connecting rod radius/length ⇒ $\lambda = \frac{R}{l}$

ω : angular velocity ⇒ $\varphi = \omega \cdot t \quad with \quad \omega = \frac{2 \cdot \pi \cdot N}{60}$

N : engine speed in revolutions per minute [1/min]



2. Thermodynamic cycle (P-v)

$$P \cdot v = r \cdot T \quad with \quad r = \frac{\bar{r}}{\tilde{m}} = \frac{\Re}{\tilde{m}}$$

$$r_{air} = \frac{\bar{r}}{\tilde{m}_{air}} = \frac{8'314}{28.97} = 287 \; J/_{kgK}$$

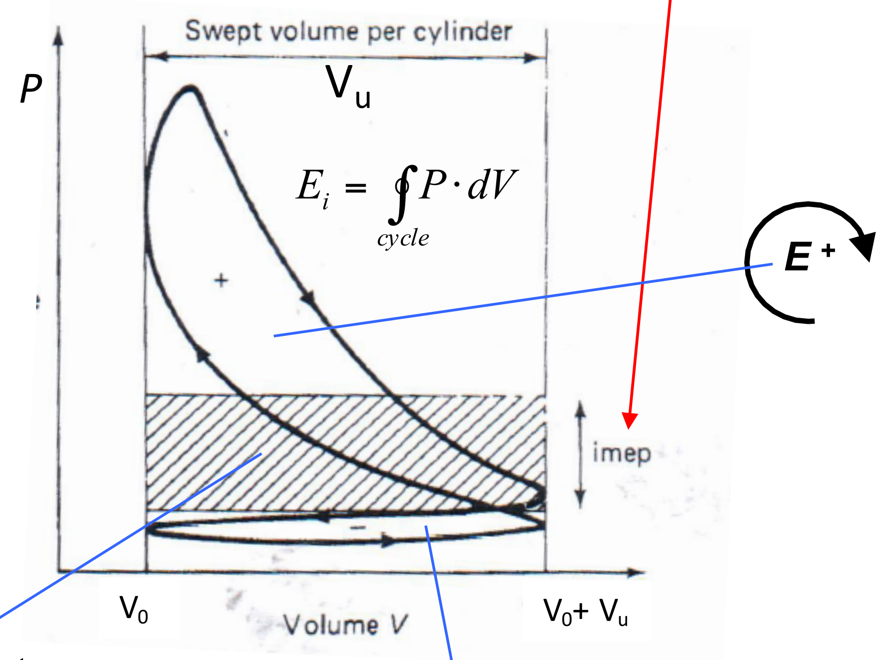
$$\gamma = C_P / C_V$$

$$\boxed{\eta_I = 1 - \frac{1}{\varepsilon^{\gamma-1}}}$$ theoretical thermodynamic efficiency (Otto)

$$\eta_I = 1 - \frac{1}{\varepsilon^{\gamma-1}}\left[\frac{\delta^{\gamma}-1}{\gamma \cdot (\delta - 1)}\right]$$ theoretical thermodynamic efficiency (Diesel)

$$with \quad \delta = \frac{V_3}{V_2} = \frac{T_3}{T_2}$$ (combustion delay)

Otto-cycle: representation
with indicated mean pressure (IMEP)



area equivalent
to net cycle surface

negative low
pressure loop

$$E_i = \oint_{cycle\,HP} P \cdot dV + \oint_{cycle\,LP} P \cdot dV = (IMEP_{HP} + IMEP_{LP}).V_u$$

$$\boxed{E_i = \oint_{cycle} P \cdot dV = IMEP \cdot V_u}$$

$$\boxed{IMEP_{LP} \approx \Delta P_{exh-adm} = P_{exh} - P_{adm}}$$

$$\boxed{IMEP = \frac{E_i}{V_u}}$$

3a. Key values : **work (E), torque (C), power ($\dot{E}$)**

C_e effective torque (Nm) : measured at the engine's outlet shaft

C_i indicated torque (Nm) : determined with the *P-v* diagram

E_i = indicated work per cycle
= torque C_i x 2 revolutions (4-stroke)

*E_i, E_e, E_f : work (J) – **i**ndicated , **e**ffective , **f**riction*

$$E_i = \oint_{cycle} P \cdot dV = \underline{IMEP \cdot V_u} = 4\pi \cdot C_i$$

*$\dot{E}_i$, $\dot{E}_e$, $\dot{E}_f$: power (W) – **i**ndicated , **e**ffective , **f**riction*

$$\dot{E}_i = \dot{E}_e + \dot{E}_f \qquad \dot{E}_f = \dot{E}_i - \dot{E}_e$$

***I**ndicated power*

$$\dot{E}_i = n_c \cdot E_i \quad with \quad n_c = \frac{N(rpm)}{60 \cdot n_{TM}} \quad and \quad E_i = \oint_{cycle} P \cdot dV$$

$$\dot{E}_i = \frac{1}{120} \cdot N(rpm) \cdot E_i$$

n_c = number of cycles per second

n_{TM} = number of revolutions for 1 Pv cycle

(for 2-Stroke n_{TM} = 1, for 4-Stroke $\boldsymbol{n_{TM} = 2}$)

***E**ffective (or Break) power*

$$\dot{E}_e = C_e \cdot \omega$$

ω in [rad/s] ; *power $\dot{E}$ = torque C x angular speed ω*

$$\dot{E}_e = C_e \cdot \frac{2\pi}{60} \cdot N$$

N in [1/min] or [rpm]

$$\dot{E}_{e,tot} = \dot{E}_{i,cyl} \cdot n_{cyl,E^+} - \dot{E}_{f,eng}$$

n_{cyl} : number of cylinders

$$IMEP = \frac{4\pi}{100} \cdot \frac{C_i}{V_u}$$

$$BMEP = \frac{4\pi}{100} \cdot \frac{C_e}{V_u}$$

IMEP in [bar]
BMEP in [bar] → 10^5 Pa
V_u in [dm³] or L → 10^{-3} m³
factor 100

$\dot{E}$ in [kW]
IMEP/BMEP in [bar]
V_u in [dm³] or L
N in [rpm]

$$\dot{E}_i = \frac{1}{1200} \cdot IMEP \cdot V_u \cdot N$$

$$\dot{E}_e = \frac{1}{1200} \cdot BMEP \cdot V_u \cdot N$$

3b. Key values : **efficiency**

Δh° = LHV in [kJ/kg]
$\dot{M}_F$ in [kg/s]
$\dot{Y}^+_{comb}$ in [kW]

Indicated efficiency : $\boxed{\eta_i = \frac{\dot{E}_i}{\dot{Y}_{comb}}}$ =without friction losses

$\dot{Y}^+_{comb} = \dot{M}_F \cdot LHV$

Effective efficiency : $\boxed{\eta_e = \frac{\dot{E}_{effective}}{\dot{Y}_{comb}}}$ =with friction losses $\boxed{\dot{E}_e = \eta_e \cdot \underbrace{LHV \cdot \dot{M}_F}_{\approx \dot{Y}^+_{comb}}}$

Mechanical efficiency : $\boxed{\eta_{mec} = \frac{\dot{E}_e}{\dot{E}_i} = \frac{\eta_e}{\eta_i}}$ expresses the friction losses $\boxed{\eta_{mec} = \frac{\dot{E}_e}{\dot{E}_i} = \frac{BMEP}{IMEP}}$

Thermodynamic (theoretical) efficiency : $\boxed{\eta_{th.th} = \frac{E^-_{ideal\ cycle}}{Q^+_{comb}}}$ => *(Otto cycle):* $\boxed{\eta_{th.th} = 1 - \frac{1}{\varepsilon^{\gamma-1}}}$

Thermal efficiency : $\boxed{\eta_t = \frac{E^-_i}{Q^+_{comb}}}$ $Q^+_F \equiv Y^+_{comb}$

Combustion efficiency: $\boxed{\eta_{comb} = \frac{Q^+_{comb}}{Q^+_F}}$

(typically 97-99%; combustion losses: 1-3%)

Cycle efficiency (shape efficiency) : $\boxed{\eta_{cycle} = \frac{E^-_i}{E^-_{ideal\ cycle}}}$

=> $\boxed{\eta_t = \eta_{th.th} \cdot \eta_{cycle}}$ Typically (example) : 42% (thermal) = 60% (Otto, theor.) x 70% (cycle)

3c. Key values : **specific fuel consumption**

Break specific fuel consumption BSFC:

$$BSFC = \frac{\left(g/h\right)}{\left(kW\right)} \rightarrow BSFC = \frac{\dot{M}_F \cdot 3600 \cdot 10^3}{\dot{E}_e}$$

$$\dot{M}_F = BSFC \cdot \dot{E}_e$$

in (kg/s).W

BSFC in [g/kWh]

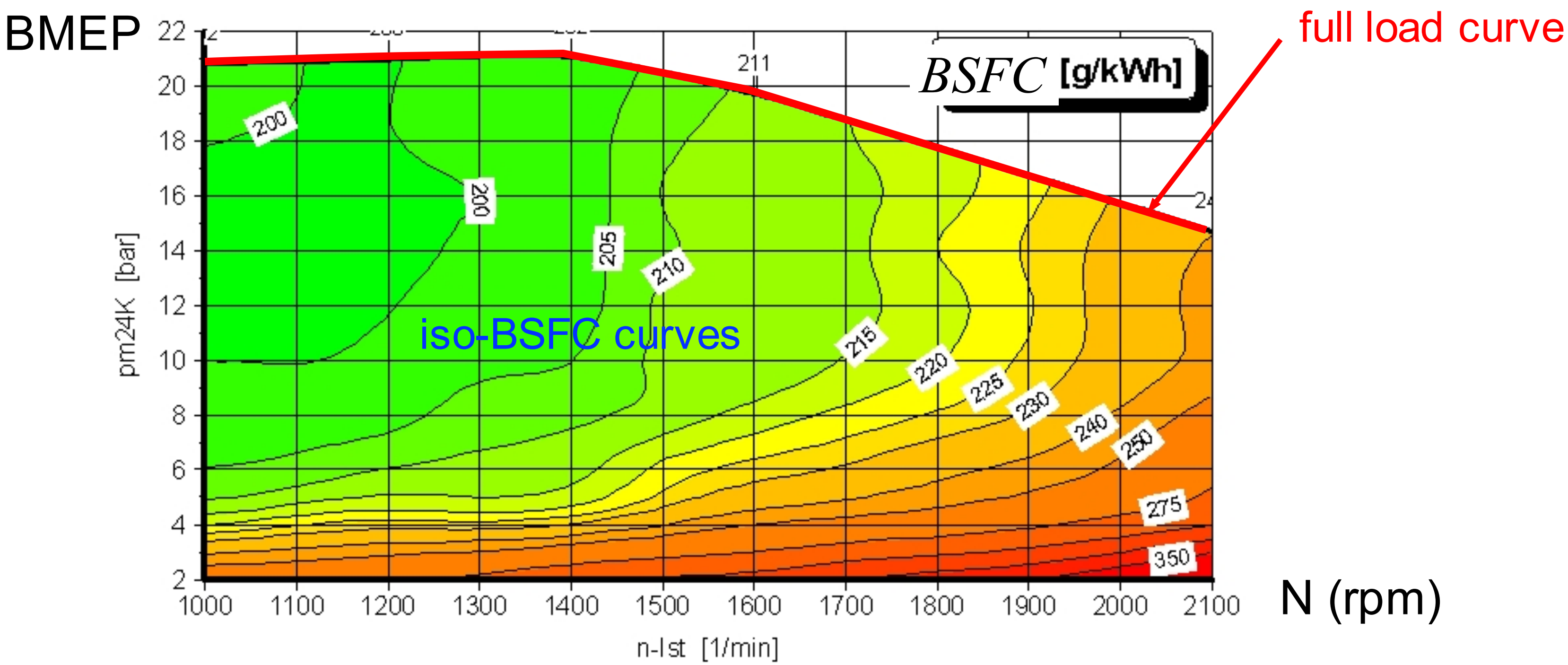
$\dot{M}_F$ in [**kg**/s]

$\dot{E}_e$ in [**kW**]

LHV in [kJ/kg]

$$\eta_e = \frac{3'600'000}{LHV \cdot BSFC}$$

$$\dot{E}_e = \eta_e \cdot \underbrace{LHV \cdot \dot{M}_F}_{\approx \dot{Y}^+_{comb}}$$



3d. Combustion

$$\boxed{R_{A/F} = \frac{M_{A,sto}}{M_F} = \frac{N_{A,sto}}{N_F} \cdot \frac{\tilde{m}_A}{\tilde{m}_F}}$$

$R_{A/F}$ mass [kg_F / kg_{Air}]

stoichiometric air/fuel ratio

$$\boxed{\tilde{R}_{A/F} = \frac{N_{A,sto}}{N_F}}$$

$R_{A/F}$ molar [$kmol_F$ / $kmol_{Air}$]

M_A in [kg of Air]

N_A in [kmol of Air]

$\tilde{m}_A$ in [kg/kmol of Air]

$$\boxed{\lambda = \frac{M_A}{M_{A,sto}} = \frac{N_A}{N_{A,sto}}}$$

Air/fuel excess ratio [kg_F / kg_{Air}]

General combustion equation:

$$N_F \cdot C_v H_w S_x O_y N_z + N_{O_2,sto} \cdot \left(O_2 + 3.762 \cdot N_2\right) \rightarrow N_{CO_2} \cdot CO_2 + N_{H_2O} \cdot H_2O + N_{SO_2} \cdot SO_2 + N_{N_2,sto} \cdot N_2$$

v,w,x,y,z : atomic stoichiometry coefficients for C, H, S, O, N in the fuel chemical formula

$$R_{A/F} = \frac{M_{A,sto}}{M_F} = \frac{N_{A,sto}}{N_F} \cdot \frac{\tilde{m}_A}{\tilde{m}_F} = \frac{4.762 \cdot N_{O_2,sto}}{N_F} \cdot \frac{\tilde{m}_A}{\tilde{m}_F} = 4.762 \cdot \left(v + \frac{w}{4} + x - \frac{y}{2}\right) \cdot \frac{28.85}{\tilde{m}_F}$$

4. Load regulation (1)

Monocylinder : $\boxed{\dot{E}_i = n_c \cdot E_i}$ and $\eta_i = \frac{\dot{E}_i}{\dot{Y}_{comb}} \approx \frac{\dot{E}_i}{\dot{M}_F \cdot LHV}$

- $\dot{E}_i$ as a function of $\boxed{M_F}$: $\boxed{\dot{E}_i = n_c \cdot \eta_i \cdot M_F \cdot LHV}$ with $\dot{M}_F = n_c \cdot M_F = n_c \cdot \boxed{\frac{M_A}{R_{A/F}} \cdot \frac{1}{\lambda}}$

  M_F : fuel quantity introduced per cycle (and per cylinder)

  n_c : number of engine cycles per second

- $\dot{E}_i$ as a function of $\boxed{M_A}$: $\boxed{\dot{E}_i = n_c \cdot \eta_i \cdot M_A \cdot \underbrace{\frac{LHV}{R_{A/F} \cdot \lambda}}_{q}}$

  M_A : air mass introduced per cylinder

  q : energy content of the mixture: [kJ/kg$_{air}$]

- M_A as a function of P_{coll}, T_{coll} : ideal gas law: $\boxed{M_A = \eta_{vol} \cdot \frac{P_{coll}}{r \cdot T_{coll}} \cdot V_u}$ or $M_A = \eta_{vol} \cdot \rho_{coll} \cdot V_u$

  η_{vol} : volumetric efficiency $\boxed{\eta_{vol} = \frac{M_{A,real}}{M_{A,ideal}}}$ *with* $M_{A,ideal} = \rho_{coll} \cdot V_u$

  V_u : cylinder displacement

=> *formula next slide*

4. Load regulation (2)

Monocylinder :

previous => slide : $\dot{E}_i$ as a function of P_{coll}, T_{coll} : $\boxed{\dot{E}_i = n_c \cdot \eta_i \cdot \eta_{vol} \cdot \frac{P_{coll}}{r \cdot T_{coll}} \cdot V_u \cdot q}$

$\dot{E}_i$ as a function of M_A : $\boxed{\dot{E}_i = \eta_i \cdot \dot{M}_A \cdot q}$ with $\boxed{\dot{M}_A = n_c \cdot \underbrace{\eta_{vol} \cdot \frac{P_{coll}}{r \cdot T_{coll}} \cdot V_u}_{M_A}}$

M_A : air mass introduced per cylinder (kg)

$\dot{M}_A$: air flow per second (kg/s)

Multicylinder : $\boxed{\dot{E}_i = \eta_i \cdot \underbrace{\frac{N}{60 \cdot n_{TM}}}_{n_c} \cdot \underbrace{\eta_{vol} \cdot \frac{P_{coll}}{r \cdot T_{coll}} \cdot V_{cyl}}_{M_A} \cdot q}$ with $V_{cyl} = n \cdot V_u$ (engine displacement)

n: number of cylinders

Effective / Break power : $\dot{E}_e = \eta_{mec} \cdot \dot{E}_i \quad \Rightarrow \quad \boxed{\dot{E}_e = \underbrace{\eta_{mec} \cdot \eta_i}_{\eta_e} \cdot \underbrace{n_c}_{N/120} \cdot M_A \cdot q}$

$$\dot{E}_e = \eta_{mec} \cdot \eta_i \cdot \dot{M}_A \cdot q \qquad \dot{E}_e = \eta_e \cdot \underbrace{\frac{N}{120} \cdot M_F}_{\dot{M}_F} \cdot LHV$$

4. Load regulation (3)

Multicylinder :

$\dot{E}_e$ as a function of $\dot{M}_A$:

$$\boxed{\dot{E}_e = \underbrace{\eta_{mec} \cdot \eta_i}_{\eta_e} \cdot \underbrace{\dot{M}_A \cdot q}_{\dot{M}_F \cdot LHV}}$$

$\dot{M}_a$ as a function of P_{coll}, T_{coll} :

$$\boxed{\dot{M}_A = n_c \cdot \underbrace{\eta_{vol} \cdot \frac{P_{coll}}{r \cdot T_{coll}} \cdot V_u}_{M_A} \cdot n}$$

with $V_{cyl} = n \cdot V_u$

n: number of cylinders

$\dot{M}_A$: total air mass flow of the engine

n_c : number of engine cycles per second

M_A : mass of air introduced into a cylinder and per cycle

q : energy content of the mixture : [kJ/kg$_{air}$]

$\dot{E}_e$ as a function of P_{coll}, T_{coll} :

$$\boxed{\dot{E}_e = \underbrace{\eta_{mec} \cdot \eta_i}_{\eta_e} \cdot \underbrace{\underbrace{\frac{N}{60 \cdot n_{TM}}}_{n_c} \cdot \underbrace{\eta_{vol} \cdot \frac{P_{coll}}{r \cdot T_{coll}} \cdot V_u}_{M_A} \cdot n}_{\dot{M}_A} \cdot q}$$

4. Balance expressed in terms of power $\dot{E}$ (kW)

Power engine

Fuel HHV: $\boxed{\dot{Y}^+_{comb} = \dot{M}_F \cdot \Delta h_i^0 + \dot{M}_{cond} \cdot q^0_{vap}}$

Balance: $\boxed{\dot{E}^-_e = \dot{Y}^+_{comb} - \dot{Q}^-_G - \dot{Q}^-_{ae} - \dot{Q}^-_{ah} - \dot{Q}^-_{ac}}$

Exhaust loss: $\dot{Q}^-_G = \dot{M}_G \cdot \hat{h}_G + \sum \dot{M}_I \cdot \Delta h^0_{iI}$

Water cooling loss: $\dot{Q}^-_{ae} = \dot{M}_e \cdot (h_{e2} - h_{e1})$

Oil cooling loss: $\dot{Q}^-_{ah} = \dot{M}_h \cdot (h_{h2} - h_{h1})$

Loss to atmosphere: $\dot{Q}^-_{ac}$

Effectiveness: $\boxed{\varepsilon = \eta_e = \dfrac{\dot{E}^-_e}{\dot{Y}^+_{comb}} = 1 - \dfrac{\dot{Q}^-_G + \dot{Q}^-_{ae} + \dot{Q}^-_{ah} + \dot{Q}^-_{ac}}{\dot{Y}^+_{comb}}}$

Cogeneration engine

Fuel transformation power: $\dot{Y}^+_{combM} = \dot{M}_F \Delta h_i^0 - \dot{M}_I \Delta h^0_{iI} + \dot{M}_{cond} q^0_{vap} - \dot{M}_G \hat{h}_G$

$\boxed{\dot{E}^-_e + \dot{Y}^-_{ae} + \dot{Y}^-_{ah} = \dot{Y}^+_{combM} - \dot{Q}^-_{ac}}$

Water transformation power: $\dot{Y}^-_{ae} = \dot{Q}^-_{ae} = \dot{M}_e (h_{e2} - h_{e1})$

Oil transformation power: $\dot{Y}^-_{ah} = \dot{Q}^-_{ah} = \dot{M}_h (h_{h2} - h_{h1})$

Effectiveness: $\boxed{\varepsilon_M = \dfrac{\dot{E}^-_e + \dot{M}_G \hat{h}_G + \dot{Y}^-_{ae} + \dot{Y}^-_{ah}}{\dot{M}_F \Delta h_i^0 - \underbrace{\left(\dot{M}_I \Delta h^0_{iI}\right)}_{\text{unburnt fuel}} + \dot{M}_{cond} q^0_{vap}}}$